8 – Exploration and Informative Planning

Dr. Marija Popović

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Information Gathering

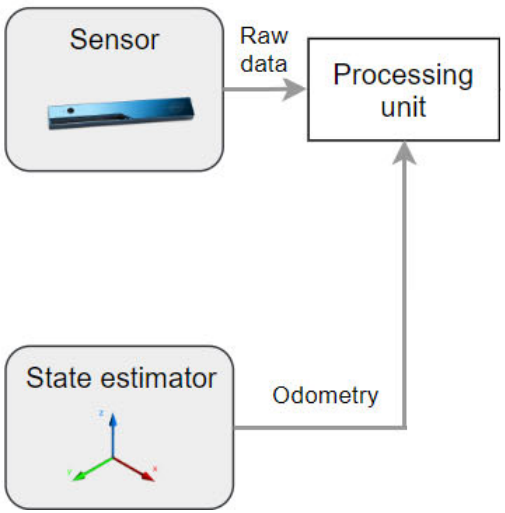
https://www.ams-samplers.com/industry/agriculture.html
https://news.rpi.edu/content/2018/08/16/jefferson-project-monitoring-harmful-algal-bloom-skaneateles-lake
https://informedinfrastructure.com/23964/gannett-fleming-prepares-for-national-tunnel-inspection-deadline-with-12-certified-inspectors
https://www.youtube.com/watch?v=5p_kqkW8jpQ
https://www.facebook.com/srfeinstein/videos/anymal-im-einsatz/1708995659165762/

Robotic Information Gathering

- **Aim**: Collect measurements to efficiently obtain an accurate model of a physical process subject to platform constraints

Hitz et al. (2014), "Fully autonomous focused exploration for robotic environmental monitoring," in: IEEE ICRA.
Bai et al. (2021), "Information-Driven Path Planning," in: Current Robotics Reports.2: 177-188.

System Diagram



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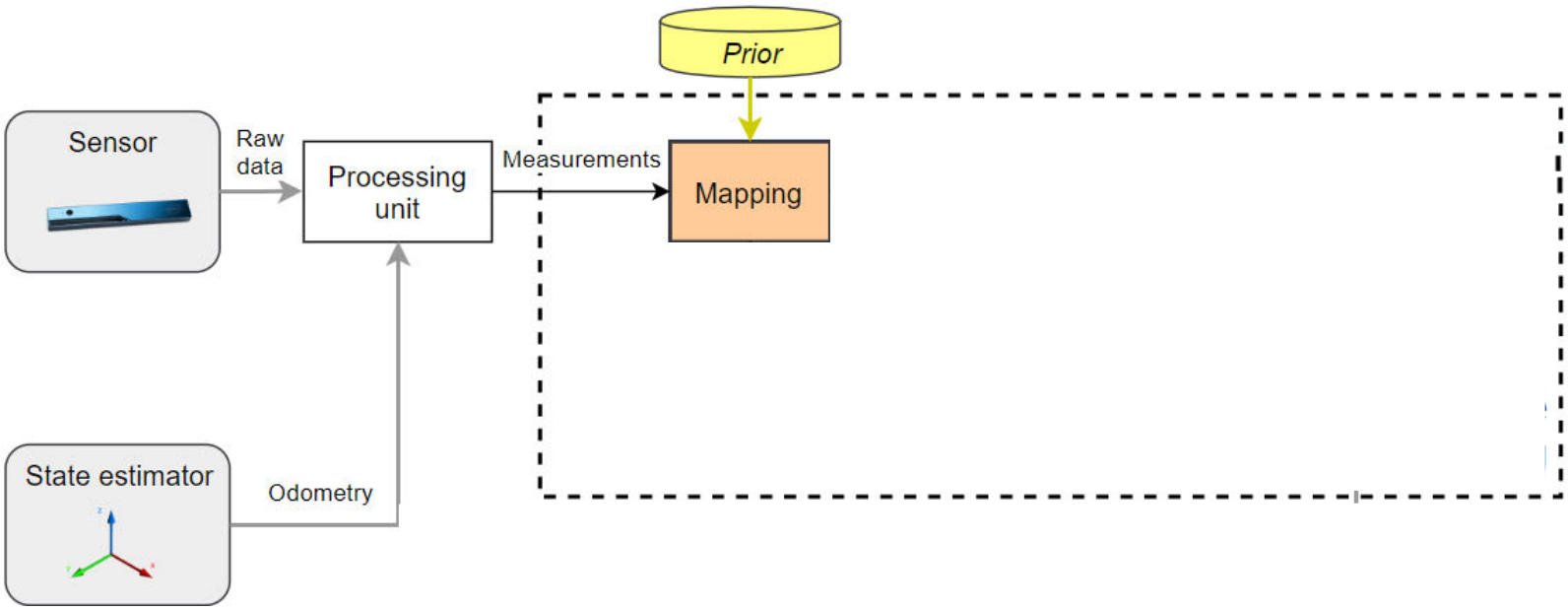

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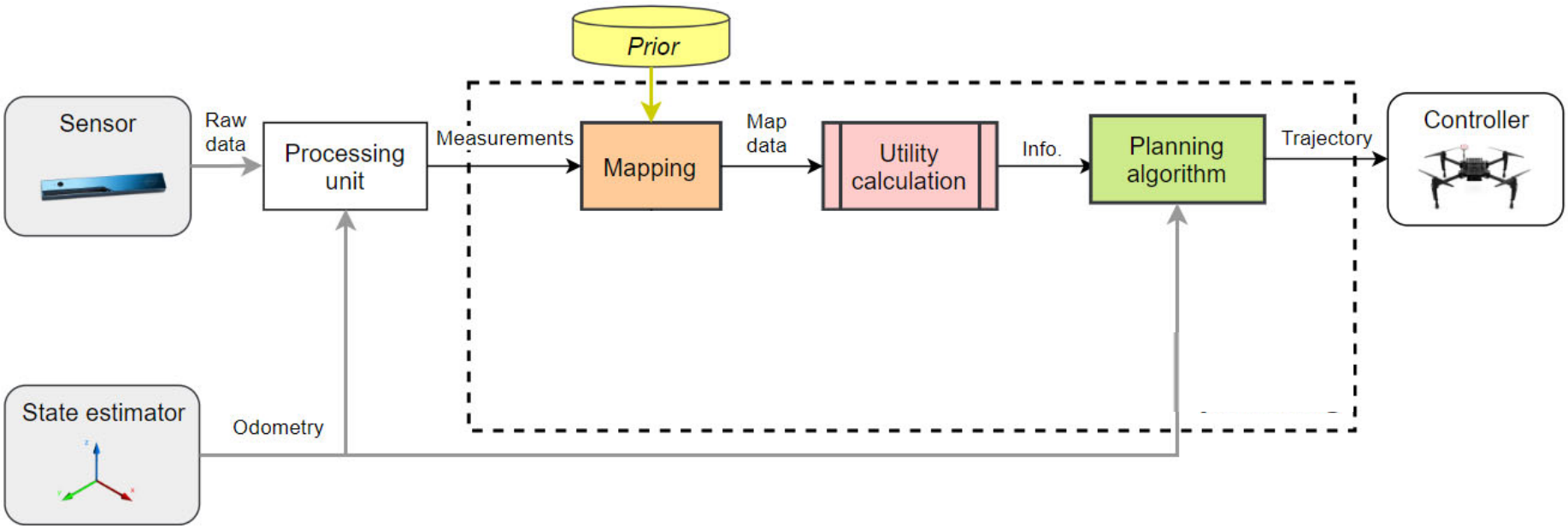
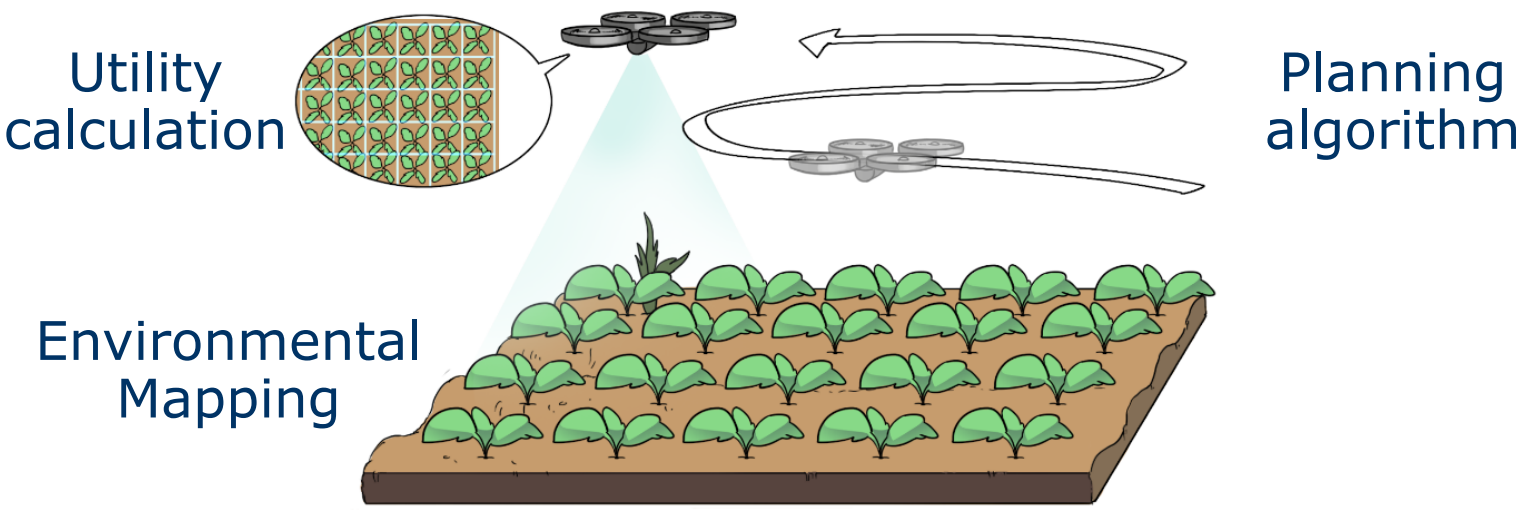


Problem Formulation

- **Aim**: Find trajectory (or path) ψ^* for max. gain in an information measure $\mathrm{I}(\cdot)$ given **cost budget** B :

$$\psi^* = \underset{\psi \in \Psi}{\operatorname{argmax}}\, \mathrm{I}(\textsc{measure}(\psi))\,,$$

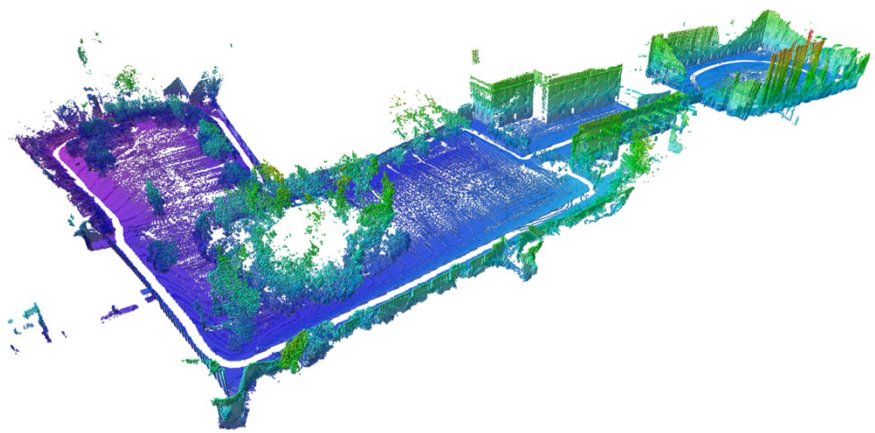
$$\text{s.t. } \textsc{cost}(\psi) \leq B\,.$$



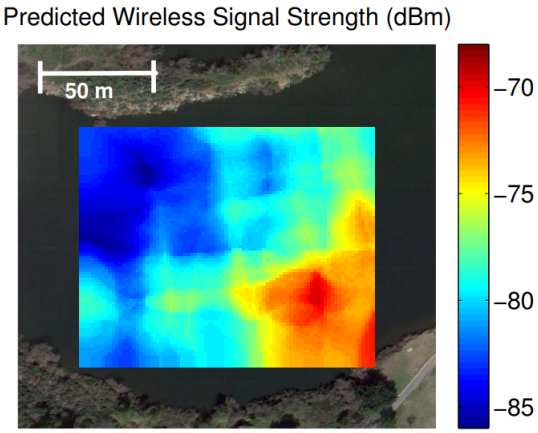
Challenges: Mapping

- How to represent the environment?
  - Dimensionality
  - Target variable: Continuous or discrete?
  - Probabilistic, multiresolution
  - Computational and memory efficiency

OctoMap



Gaussian Process



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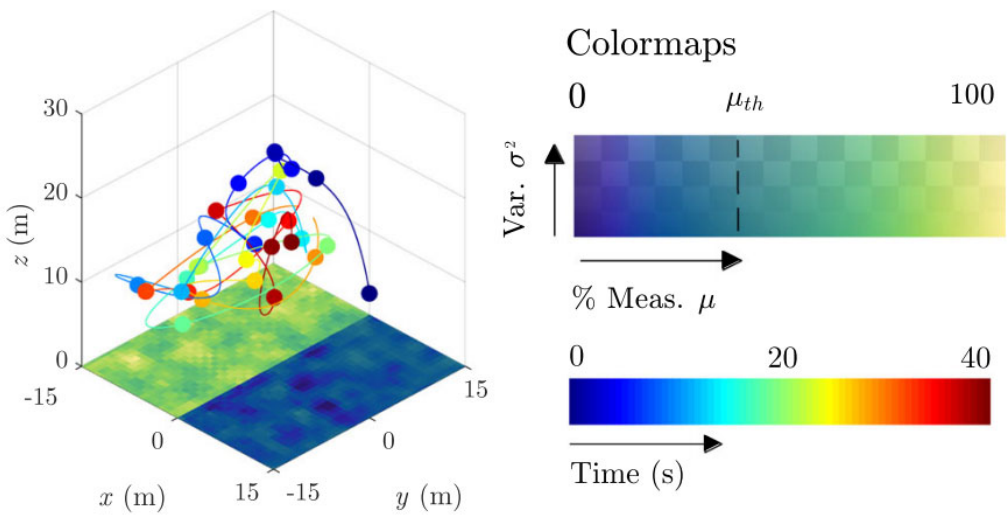
Hollinger and Sukhatme (2014), “Sampling-based robotic information gathering algorithms,“ in: The International Journal of Robotics Research. 33(9): 1271-1287.

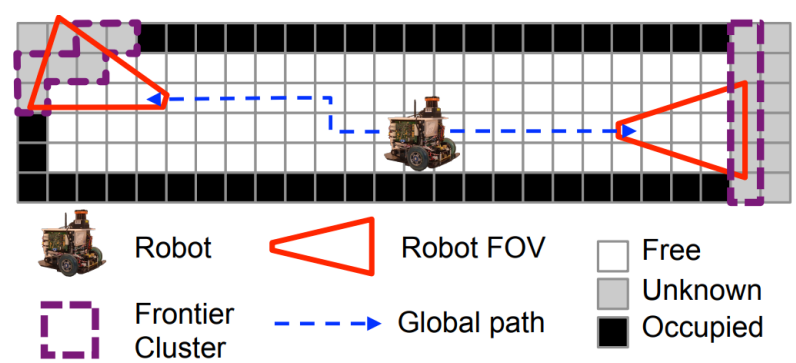
Challenges: Utility Calculation

- How to measure "usefulness"?
  - Information-theoretic measure
  - Adaptivity
  - Predictive sensor model

Adaptivity



Sensor model

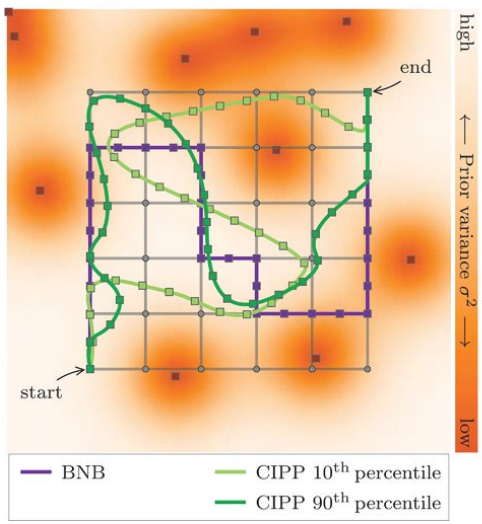


Popović et al. (2020), "An informative path planning framework for UAV-based terrain monitoring," in: Autonomous Robots. 44: 889-911.
Charrow et al. (2011), "Information-Theoretic Planning with Trajectory Optimization for Dense 3D Mapping," in: Robots: Science and Systems.

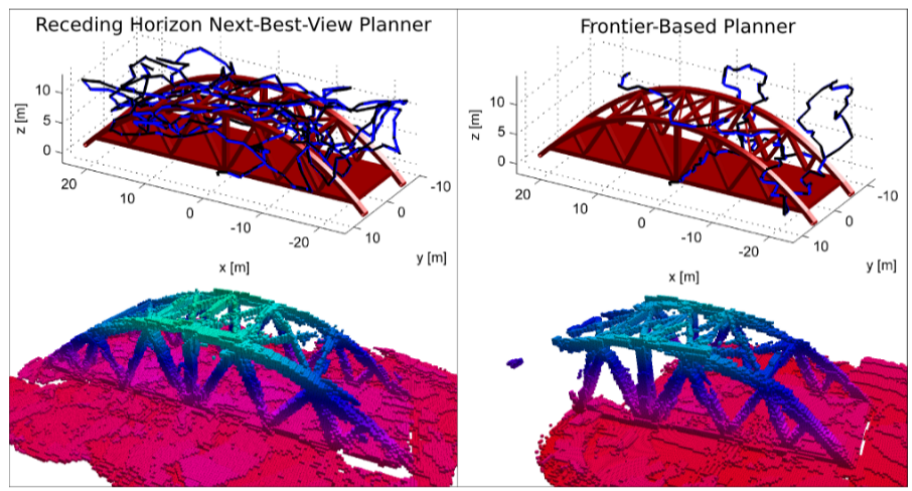
Challenges: Planning Algorithm

- How to plan informative paths?
  - Discrete vs. continuous
  - Lookahead: myopic vs. non-myopic
  - Online replanning
  - Computational and memory efficiency

Discrete vs. continuous



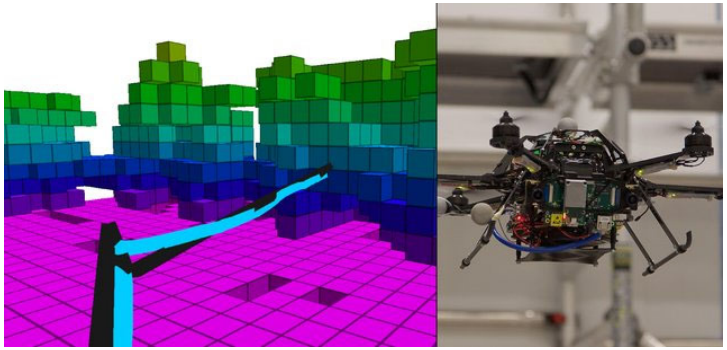
Lookahead, runtime



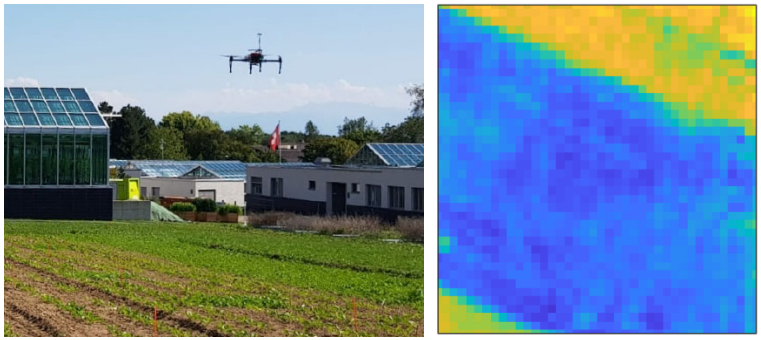
Hitz et al. (2017), "Adaptive continuous-space informative path planning for online environmental monitoring," in: Journal of Field Robotics. 34(8): 1427-1449.
Bircher et al. (2016), "Receding Horizon "Next-Best-View" Planner for 3D Exploration," in: IEEE ICRA.

Problem Variations

- **Robotic exploration**: Create a map of an unknown environment as quickly as possible.



- **Data collection planning**: Determine a cost-efficient path to collect measurements using a sensor.



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Popović et al. (2020), “An informative path planning framework for UAV-based terrain monitoring,“ in: Autonomous Robots. 44: 889-911.

Problem Variations

- **Inspection planning**: Determine a cost-efficient path to inspect a known environment.
- **Persistent monitoring**: Take regular measurements in a changing environment.
- **Active Simultaneous Localisation and Mapping (SLAM)**: Determine paths to simultaneously build a map and localise within it.

Robotic Exploration

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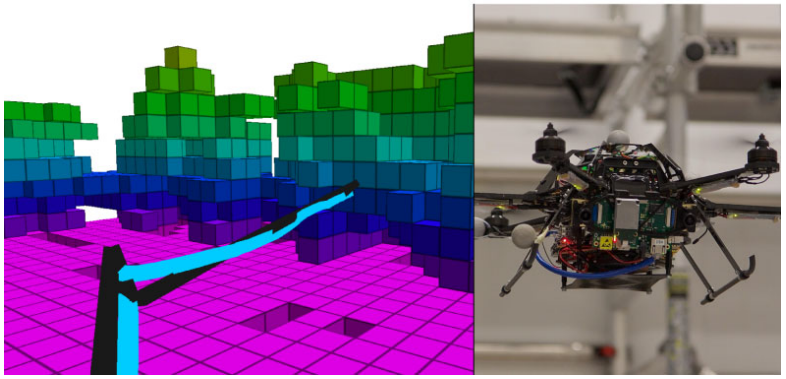
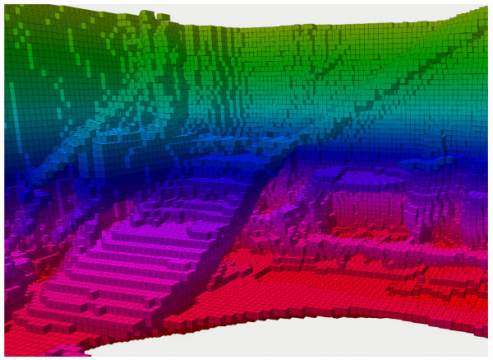
Robotic Exploration

- **Aim**: Create a map of a partially or fully unknown environment as quickly as possible
- e.g. industrial inspection, search and rescue, tunnel mapping, etc.





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Dai et al. (2020), “Fast Frontier-based Information-driven Autonomous Exploration with an MAV,“ in: IEEE ICRA. 9570-9576.

Problem Setup

- Robot equipped with sensor, e.g. stereo camera, laser scanner
- Occupancy grid map representation
- Performance metrics: exploration time, map quality

- Consider bounded 3D space $V \subset \mathbb{R}^3$
- Partition V into $V_{free} \subset V$ and $V_{occ} \subset V$ such that $V_{free} \cup V_{occ} = V \setminus V_{res}$ where V_{res} is the residual space

Bircher et al. (2016), “Receding Horizon “Next-Best-View” Planner for 3D Exploration,“ in: IEEE ICRA.

Occupancy Grid Map

- Divide environment into cells
- Each cell is a random binary variable representing occupancy
- Assumptions:
  - Area of a cell is either completely free or occupied
  - Cells are independent of each other
  - State is static
  - Robot pose known
- Probability distribution of the map:

$$p(m) = \prod_i p(m_i)$$

Bircher et al. (2016), “Receding Horizon “Next-Best-View” Planner for 3D Exploration,“ in: IEEE ICRA.

Occupancy Mapping Algorithm

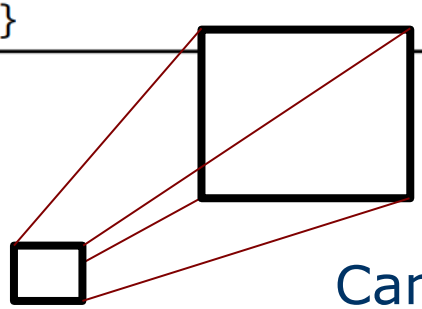
- Use binary Bayes filter to update map with new measurements

- Log-odds notation:

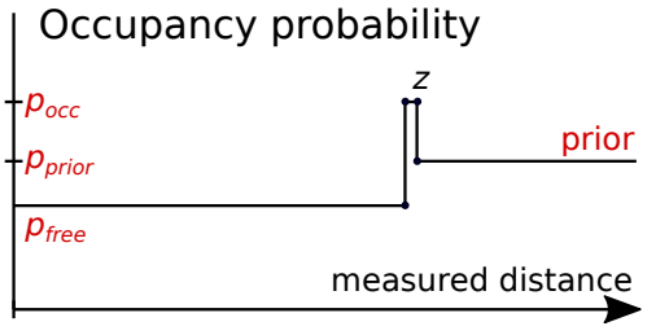
Algorithm 1: OccupancyGridMapping($\{l_{t-1,i}\}, x_t, z_t$)

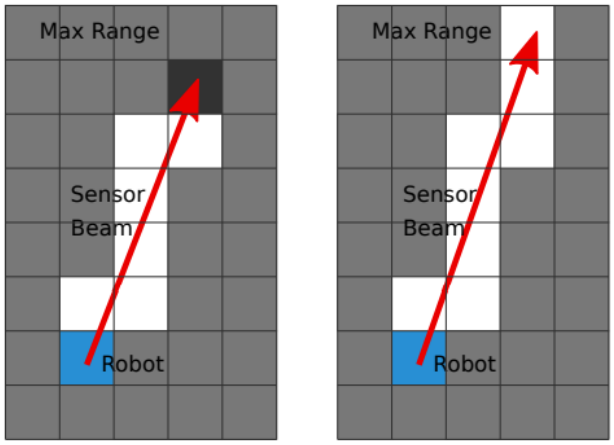
foreach m_i *of the map* m **do**
  if m_i *in the perceptual field of* z_t **then**
    $l_{t,i} := l_{t-1,i} + \boxed{\text{inv\_sensor\_model}(m_i, x_t, z_t)} - l_0$;
  else
    $l_{t,i} := l_{t-1,i}$;

return $\{l_{t,i}\}$



Camera





Laser scanner

https://cw.fel.cvut.cz/b201/_media/courses/b4m36uir/lectures/b4m36uir-lec04-handout.pdf

Occupancy Mapping Algorithm

- Use Binary bayes filter to update map with new measurements

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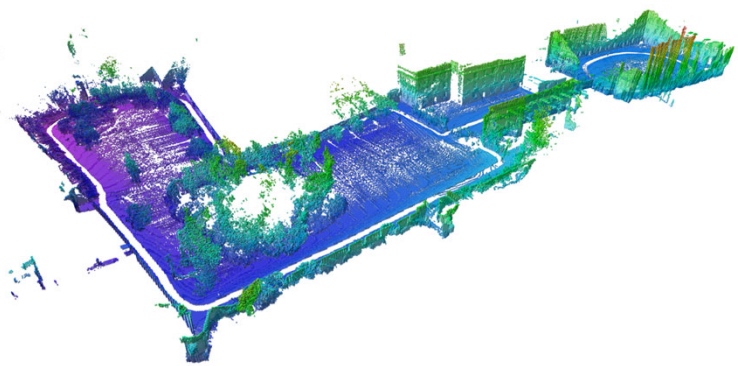
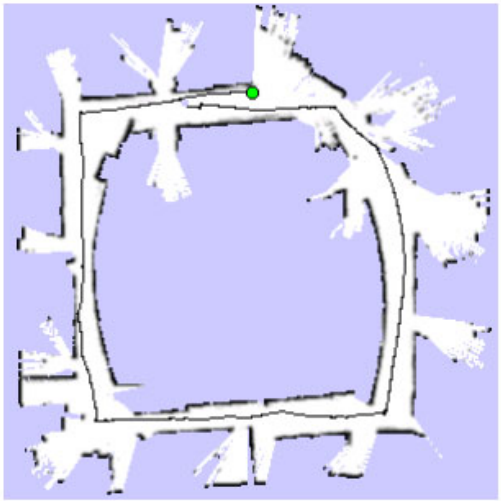
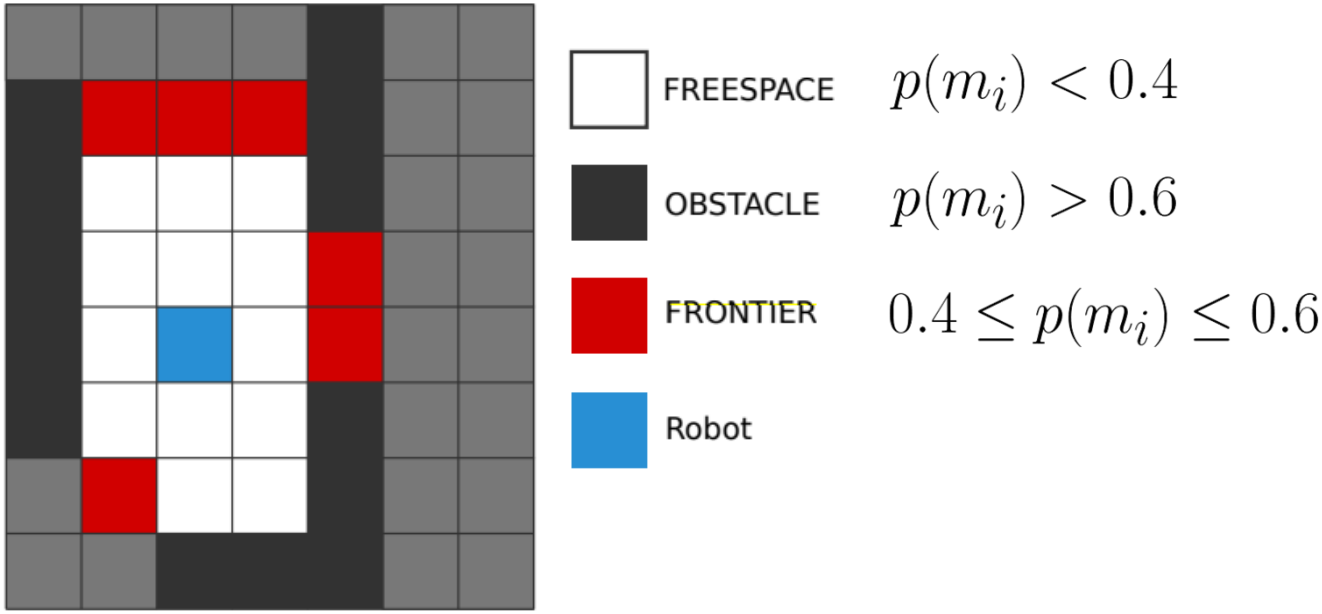




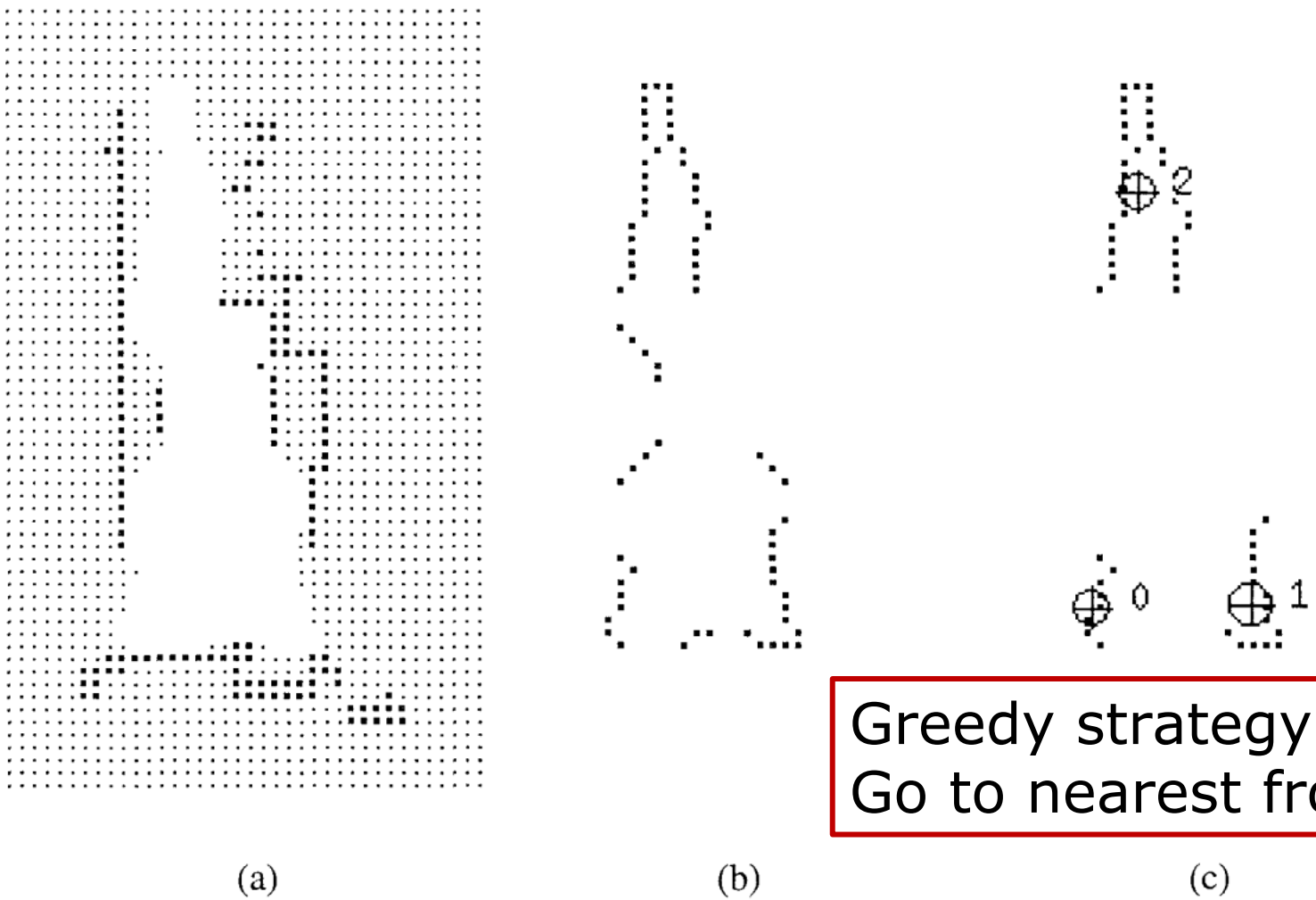
Frontier-based Exploration

- **Idea**: Move robot towards unknown regions
- **Frontier**: Border between free and unknown space in an environment



Yamauchi (1997), "A frontier-based approach for autonomous exploration," in: IEEE CIRA.
https://cw.fel.cvut.cz/b201/_media/courses/b4m36uir/lectures/b4m36uir-lec04-handout.pdf

Frontier-based Exploration

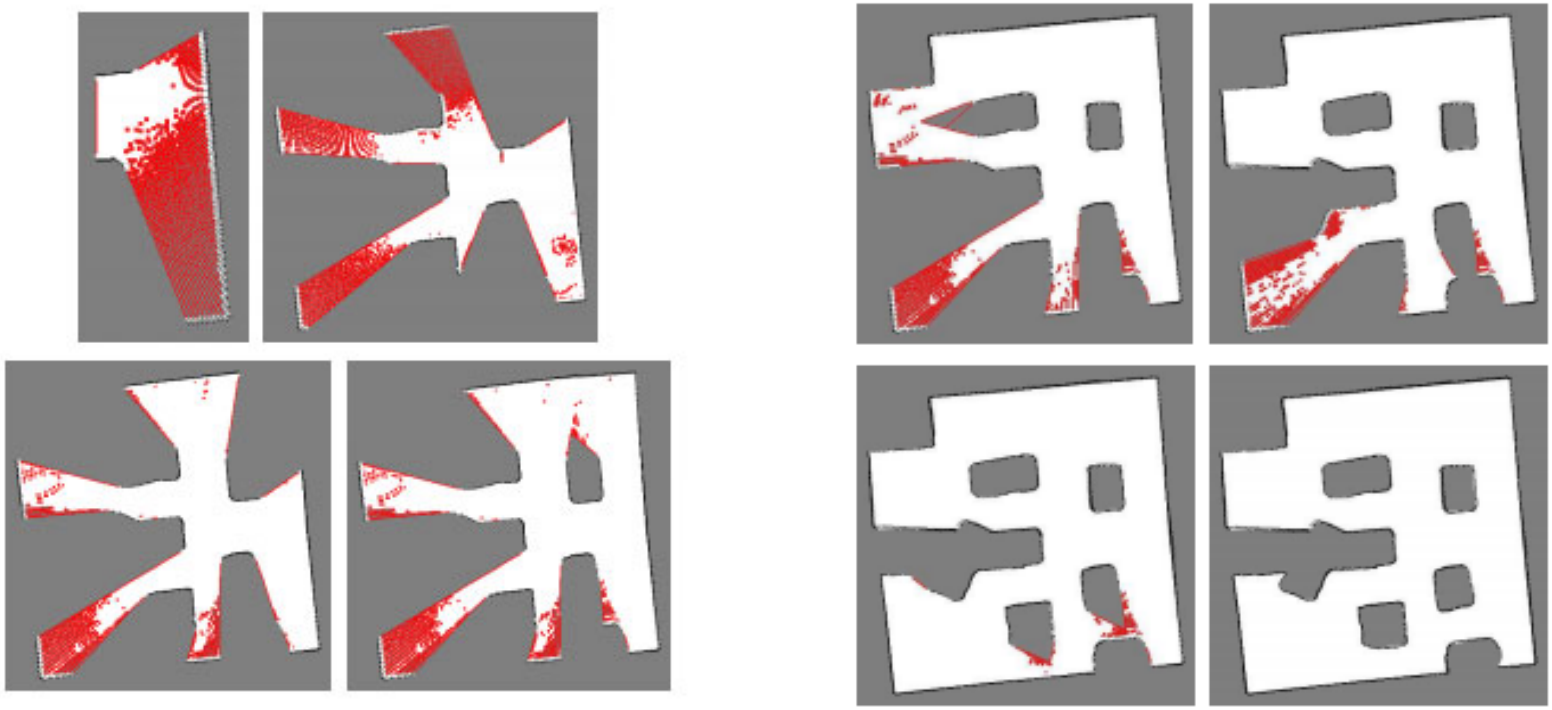


Greedy strategy:
Go to nearest frontier

Figure 1: Frontier detection: (a) evidence grid, (b) frontier edge segments, (c) frontier regions

Yamauchi (1997), “A frontier-based approach for autonomous exploration,“ in: IEEE CIRA.

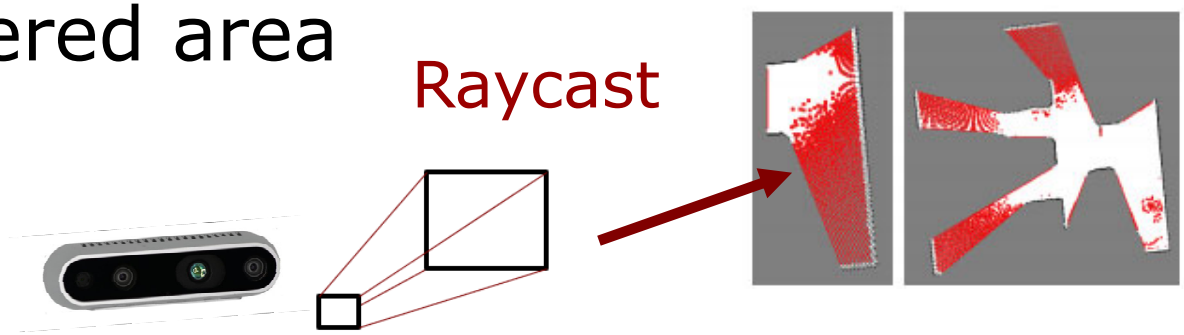
Frontier-based Exploration



https://www.ais.uni-bonn.de/~holz/papers/holz_2010_informatiktage.pdf

Improving Frontier-based Exploration

- Utility function based on expected covered area



- Utility function based on information theory

Shannon's entropy

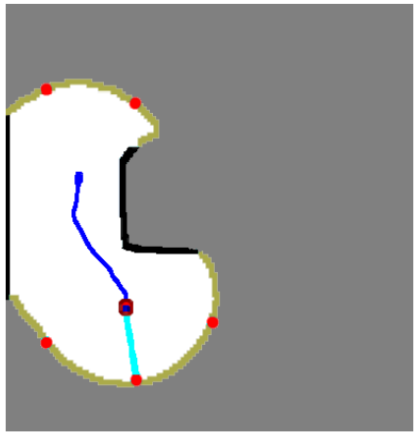
$$\mathrm{H} = -\sum_{m_i} p(m_i)\log p(m_i) + (1 - p(m_i))\log(1 - p(m_i))$$

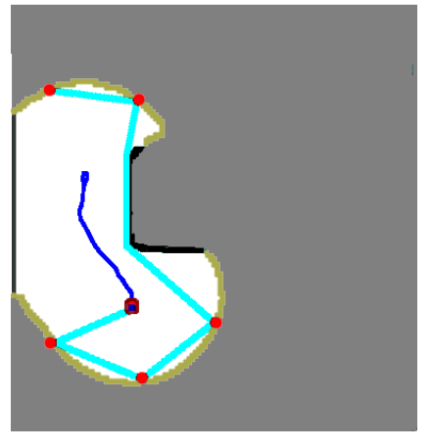
Utility/info. gain $\mathrm{I} = \mathrm{H}^- - \mathrm{H}^+$

González-Baños and Latombe (2002), "Navigation strategies for exploring indoor environments," in: IJRR. 21(10-11): 829-848.

Improving Frontier-based Exploration

- Planning algorithm with look-ahead





Travelling Salesman Problem (TSP)

- Utility function accounting for distance/time

Information gain rate

$$\psi^* = \underset{\psi \in \Psi}{\operatorname{argmax}} \frac{\mathrm{I}(\mathrm{MEASURE}(\psi))}{\mathrm{COST}(\psi)}$$

Exponential factor

$$\psi^* = \mathrm{I}(\mathrm{MEASURE}(\psi)) e^{\lambda \mathrm{COST}(\psi)}$$

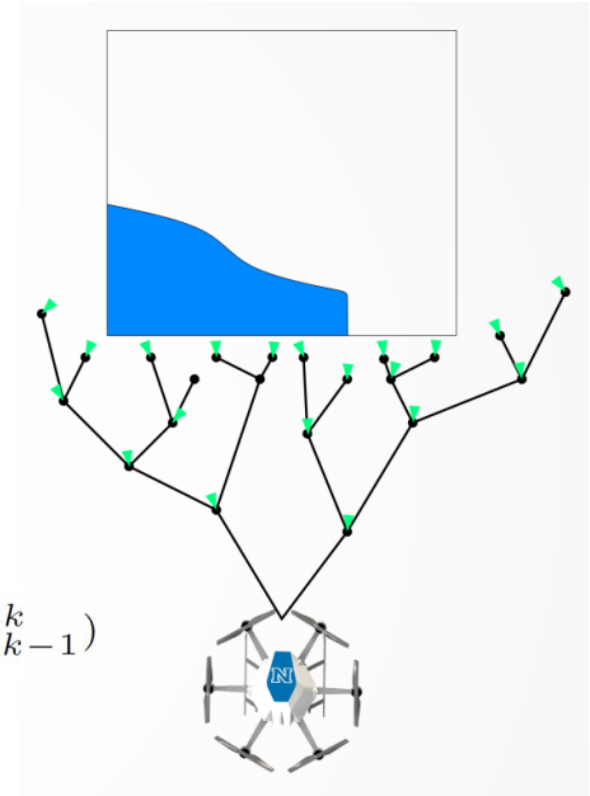
https://cw.fel.cvut.cz/b201/_media/courses/b4m36uir/lectures/b4m36uir-lec04-handout.pdf

Sampling-based Exploration

- Tree-based planning for exploration
  - Bircher et al. (2016), “Receding Horizon “Next-Best-View” Planner for 3D Exploration,“ in: IEEE ICRA.

- Create random tree in initially unknown environment
- Store utility/information gain at each node:

$$\mathbf{Gain}(n_k) = \mathbf{Gain}(n_{k-1}) + \mathbf{Visible}(\mathcal{M}, \xi_k) e^{-\lambda c(\sigma_{k-1}^k)}$$



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Sampling-based Exploration

- Tree-based planning for exploration
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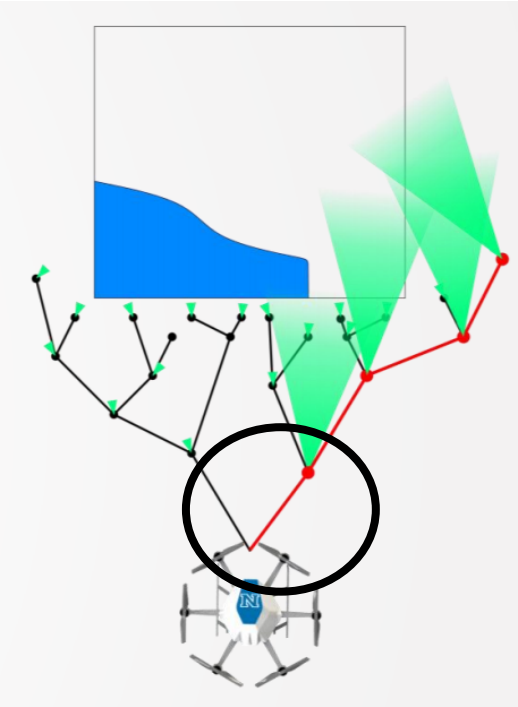
- Keep track of node highest information gain
- Receding-horizon: Execute first segment of path toward this node

$$\mathbf{ExtractBestPathSegment}(n_{best})$$



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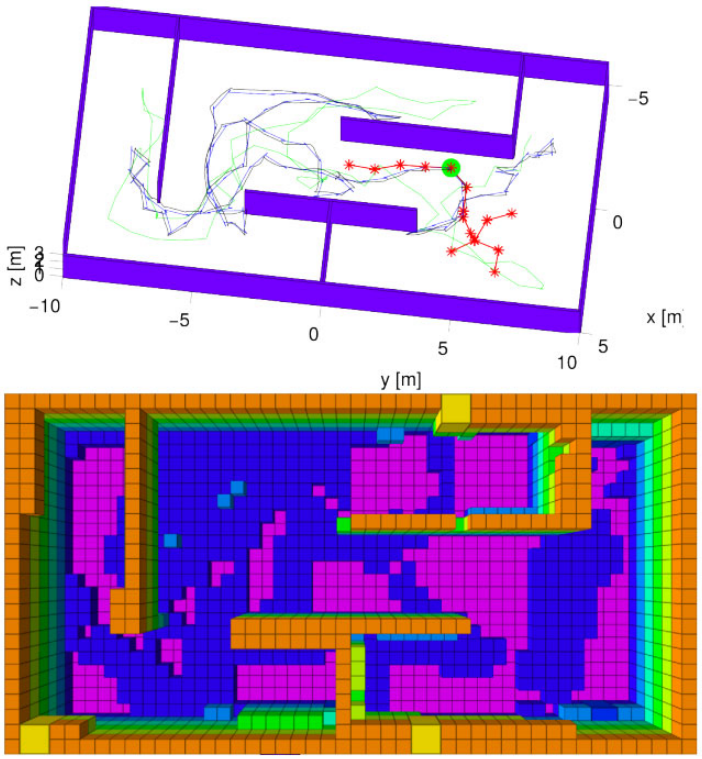
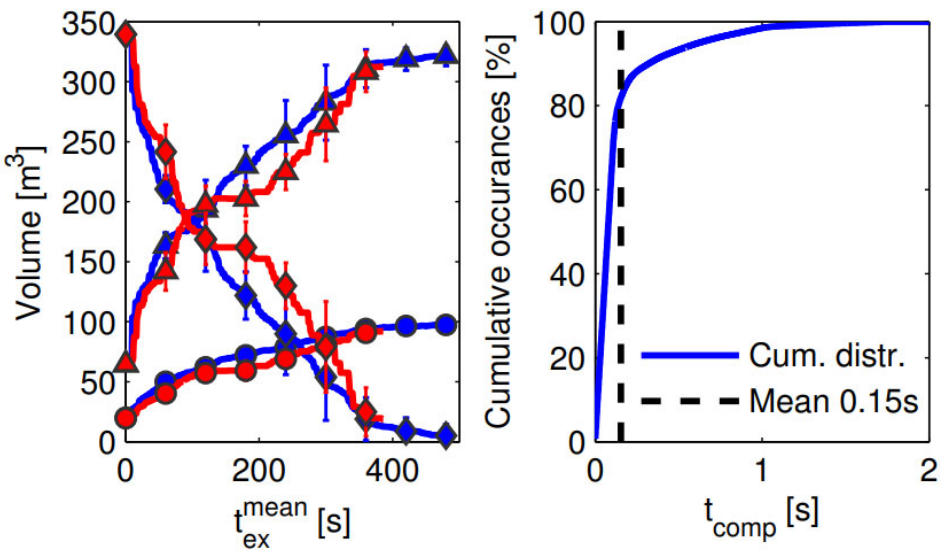
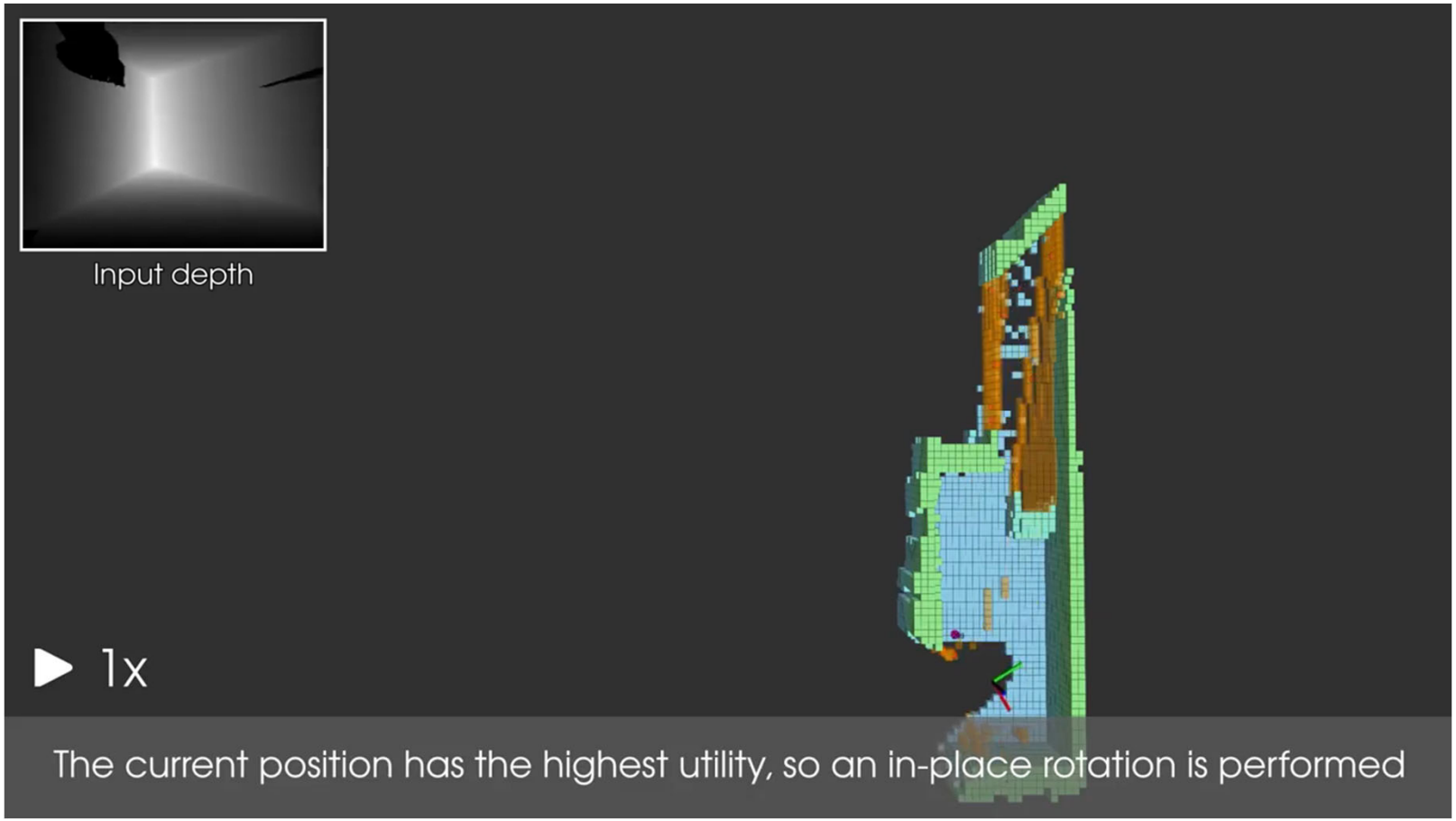




occupied o, **free** △ and **unmapped** ◊

Fast Frontier-based Exploration

- Dai et al. (2020), “Fast Frontier-based Information-driven Autonomous Exploration with an MAV,“ in: IEEE ICRA. pp.9570-9576.



Robotic Data Collection

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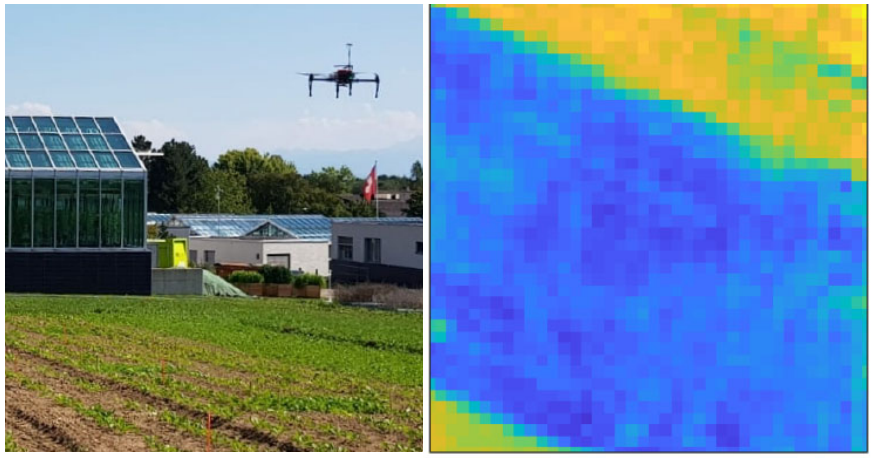
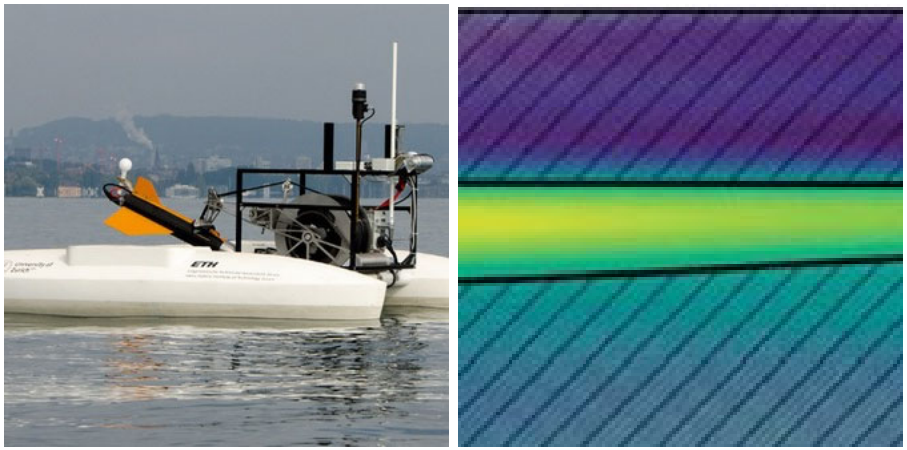
Robotic Data Collection

- **Aim**: Determine a cost-efficient path to collect measurements using a sensor
- e.g. precision agriculture, lake monitoring, temperature mapping, etc.





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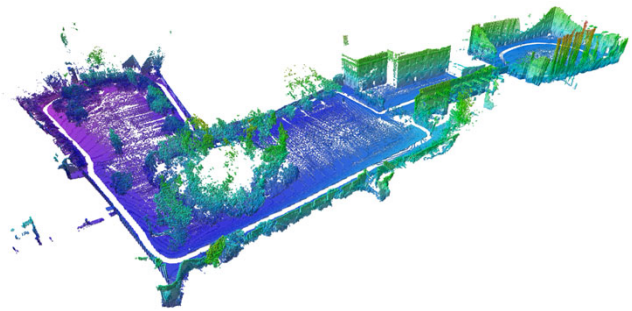
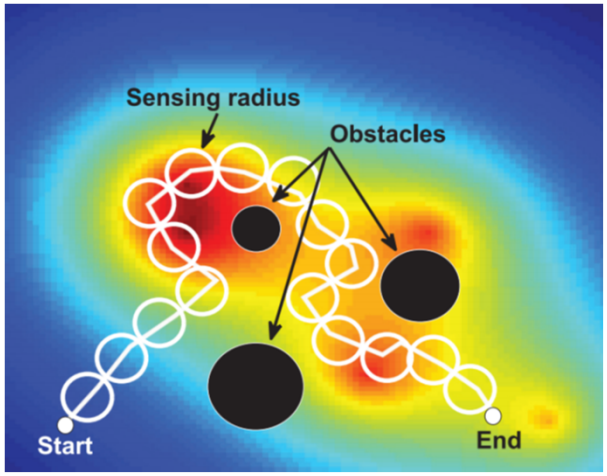
Hitz et al. (2017), “Adaptive continuous-space informative path planning for online environmental monitoring,“ in: Journal of Field Robotics. 34(8): 1427-1449.

Exploration vs. Environmental Data Collection

- **Exploration**: Plan paths and gather information in the **same** map



- **Environmental data collection**: Plan paths and gather information in **different** maps



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Hollinger and Sukhatme (2014), “Sampling-based robotic information gathering algorithms,“ in: The IJRR. 33(9): 1271-1287.

Gaussian Processes (GPs)

- Most environmental phenomena vary smoothly in time and space
- Model using a **GP**: distribution over functions fully captured by mean $\boldsymbol{\mu}$ and covariance $\mathbf{P}$
- A **kernel** captures correlations in the data

- **GP regression**:

Mean $\boldsymbol{\mu} = \mathbb{E}[\mathbf{f}_*] = m(\mathbf{X}_*) + K(\mathbf{X}_*, \mathbf{X})[K(\mathbf{X}, \mathbf{X}) + \sigma_n^2 \mathbf{I}_n]^{-1} (\mathbf{y} - m(\mathbf{X}))$

Covariance $\mathbf{P} = \mathbb{V}[\mathbf{f}_*] = K(\mathbf{X}_*, \mathbf{X}_*) - K(\mathbf{X}_*, \mathbf{X})[K(\mathbf{X}, \mathbf{X}) + \sigma_n^2 \mathbf{I}_n]^{-1} K(\mathbf{X}_*, \mathbf{X})^\top$

GP Example

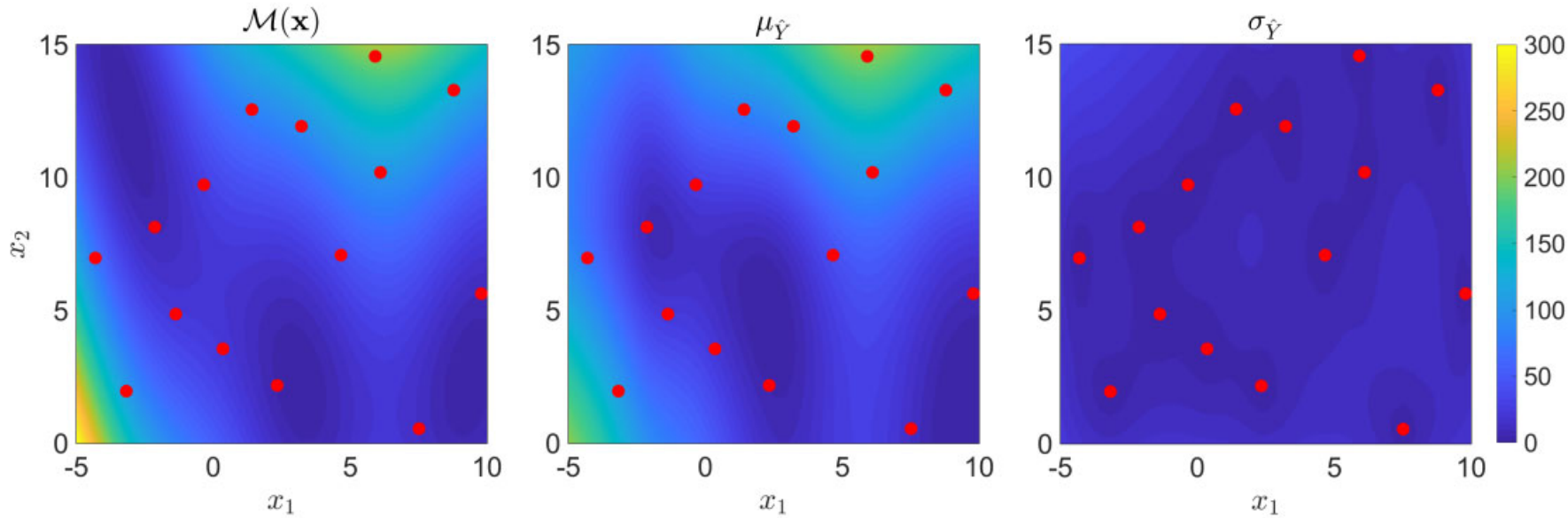


Figure 5: From left to right: the Branin-Hoo function (true model) followed by the mean and standard deviation of the Kriging predictor. The experimental design is illustrated by red dots.

RSUQ-2017-013b.pdf (ethz.ch)

Information Criteria in GPs

- Derived for the **sensor placement** problem
  - Krause et al. (2008), "Near-Optimal Sensor Placements in Gaussian Processes," in: Jour. of Machine Learning Research. 9: 235-284.
  - Entropy:

Possible sensor measurements

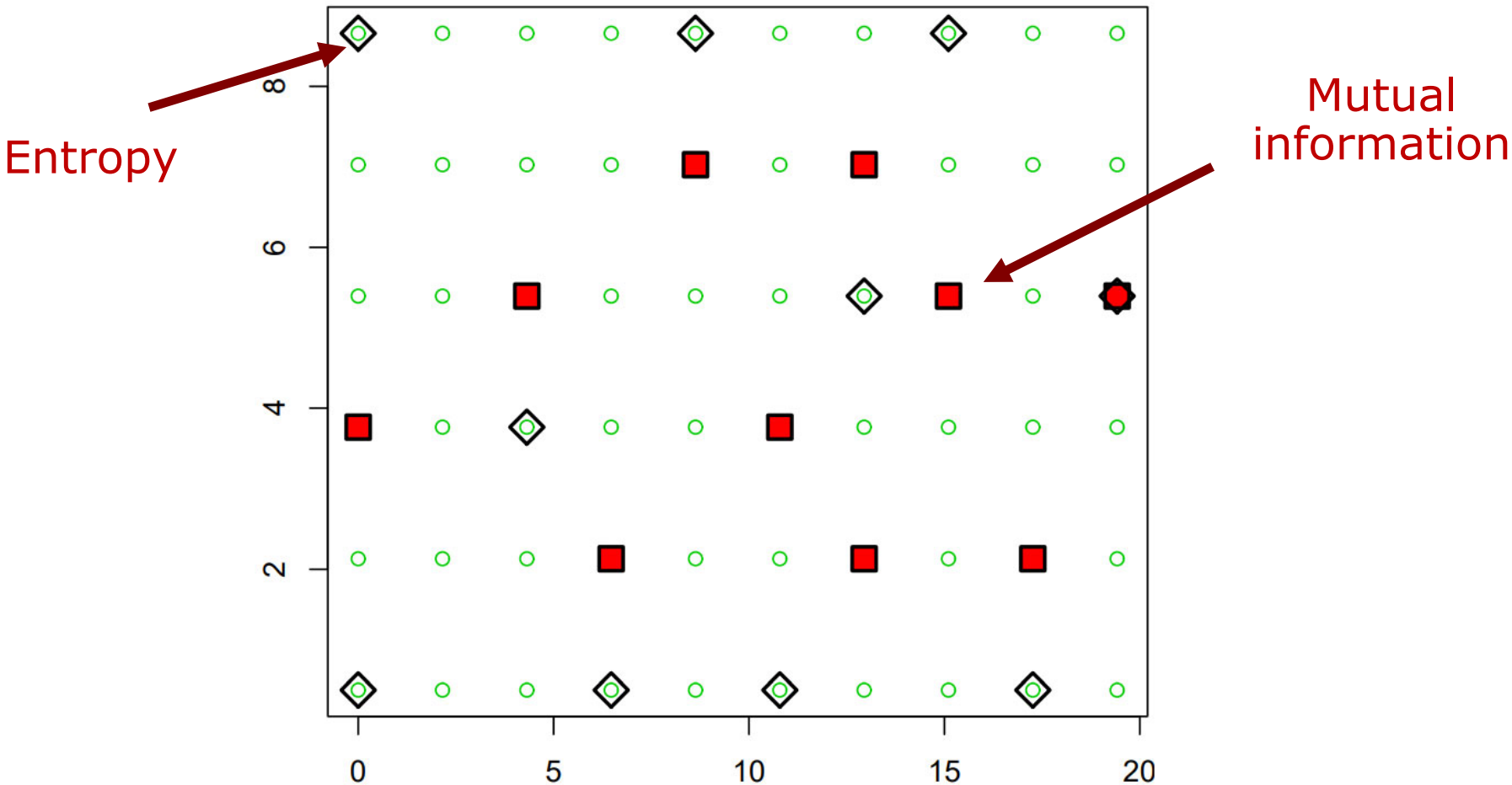
$$H(X_{\mathcal{V}\setminus\mathcal{A}} \mid X_{\mathcal{A}}) = -\int p(\mathbf{x}_{\mathcal{V}\setminus\mathcal{A}}, \mathbf{x}_{\mathcal{A}}) \log p(\mathbf{x}_{\mathcal{V}\setminus\mathcal{A}} \mid \mathbf{x}_{\mathcal{A}}) d\mathbf{x}_{\mathcal{V}\setminus\mathcal{A}} d\mathbf{x}_{\mathcal{A}}$$

$$\mathcal{A}^* = \operatorname{argmin}_{\mathcal{A}\subset\mathcal{V}:|\mathcal{A}|=k} H(X_{\mathcal{V}\setminus\mathcal{A}} \mid X_{\mathcal{A}})$$

  - Mutual information:

$$\mathcal{A}^* = \operatorname{argmax}_{\mathcal{A}\subseteq\mathcal{S}:|\mathcal{A}|=k} H(X_{\mathcal{V}\setminus\mathcal{A}}) - H(X_{\mathcal{V}\setminus\mathcal{A}} \mid X_{\mathcal{A}})$$

Information Criteria in GPs



Krause et al. (2008), “Near-Optimal Sensor Placements in Gaussian Processes,“ in: Jour. of Machine Learning Research. 9: 235-284.

Submodularity

- These objectives are **submodular**: amount of information gathered in the future depends on the prior trajectory

  For all $\mathcal{A}, \mathcal{B} \subseteq \mathcal{V}$,

  $$F(\mathcal{A}) + f(\mathcal{B}) \geq f(\mathcal{A} \cap \mathcal{B}) + f(\mathcal{A} \cup \mathcal{B})$$

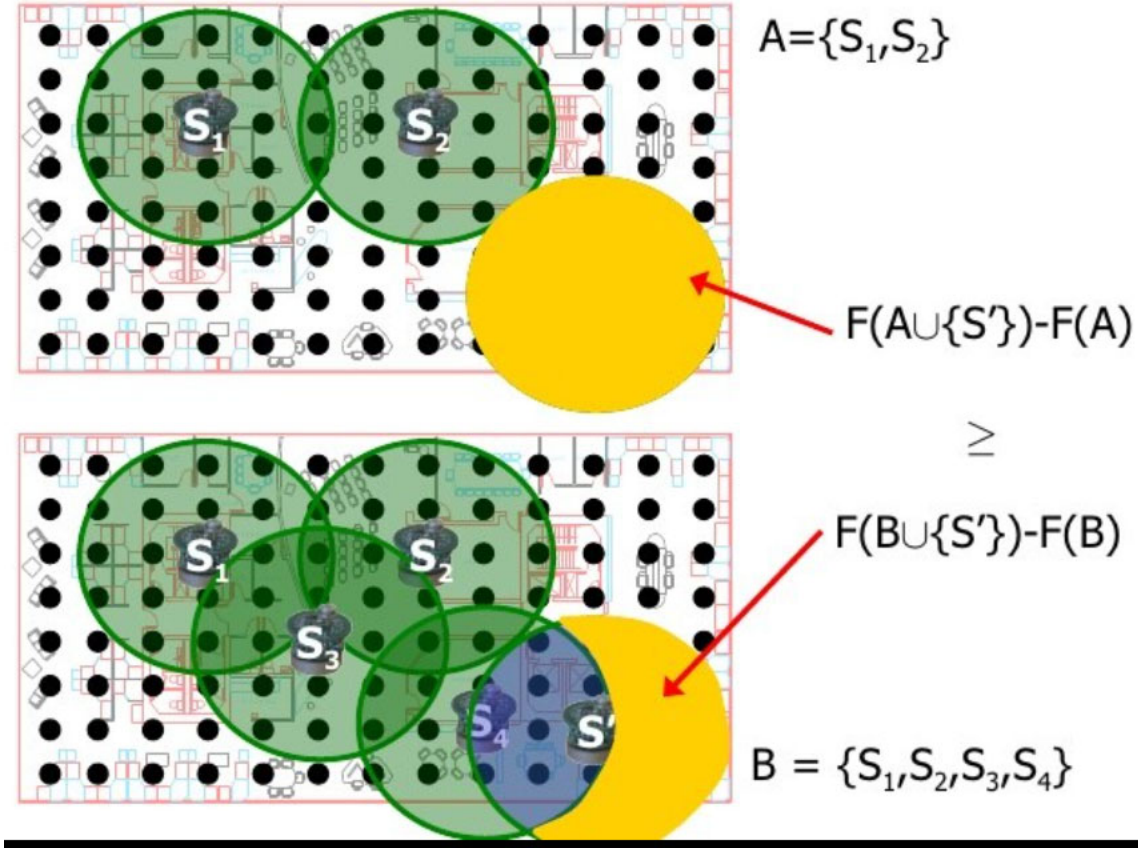
- Diminishing returns formulation:

  For $A \subseteq B, A \notin B$,

  $$F(\mathcal{A} \cup s) - F(\mathcal{A}) \geq F(\mathcal{B} \cup s) - F(\mathcal{B})$$

Krause et al. (2008), “Near-Optimal Sensor Placements in Gaussian Processes,“ in: Jour. of Machine Learning Research. 9: 235-284.

Submodularity



For $A \subseteq B, A \notin B$,

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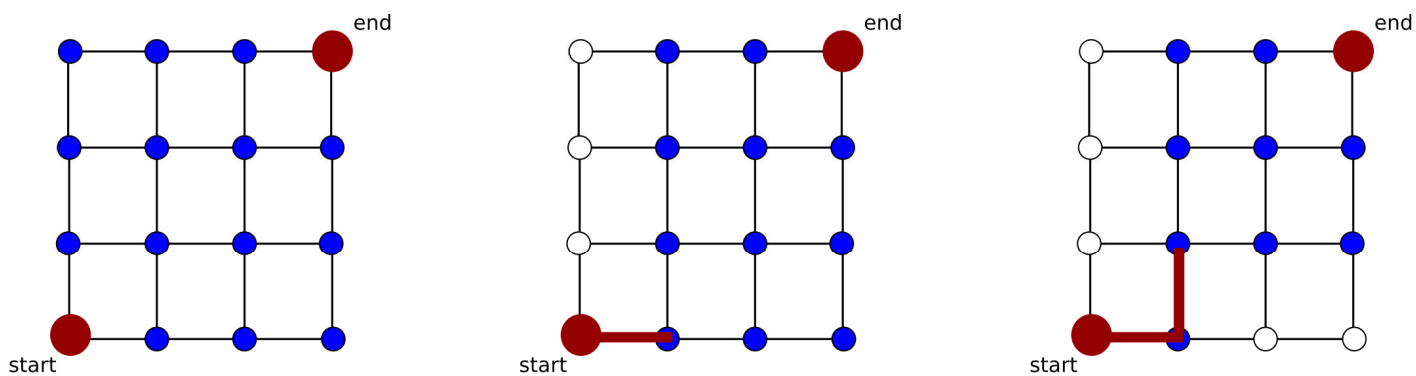
Krause et al. (2008), "Near-Optimal Sensor Placements in Gaussian Processes," in: Jour. of Machine Learning Research. 9: 235-284.

Planning Strategies

- Recursive greedy
  - "Next-best" sensing location
  - Within $(1 - 1/e)$ of optimum
- Discrete-space methods
  - Branch and bound



(a) Initialization: start and end specified, planning commences.

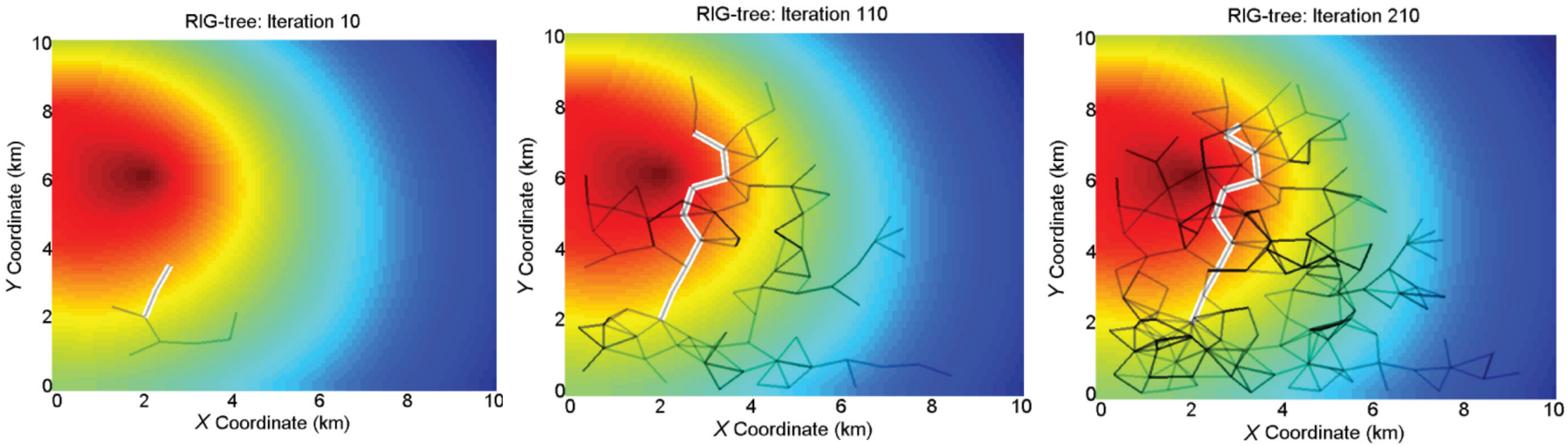
(b) After recursing once.

(c) After recursing twice.

Krause et al. (2008), "Near-Optimal Sensor Placements in Gaussian Processes," in: Jour. of Machine Learning Research. 9: 235-284.
Binney (2012). "Branch and bound for informative path planning," in: IEEE ICRA. 2147-2154.

Planning Strategies

- Continuous-space methods
  - Sampling-based: rapidly-information gathering tree (RIG-tree)



Hollinger and Sukhatme (2014), "Sampling-based robotic information gathering algorithms," in: The IJRR. 33(9): 1271-1287.

Planning Strategies

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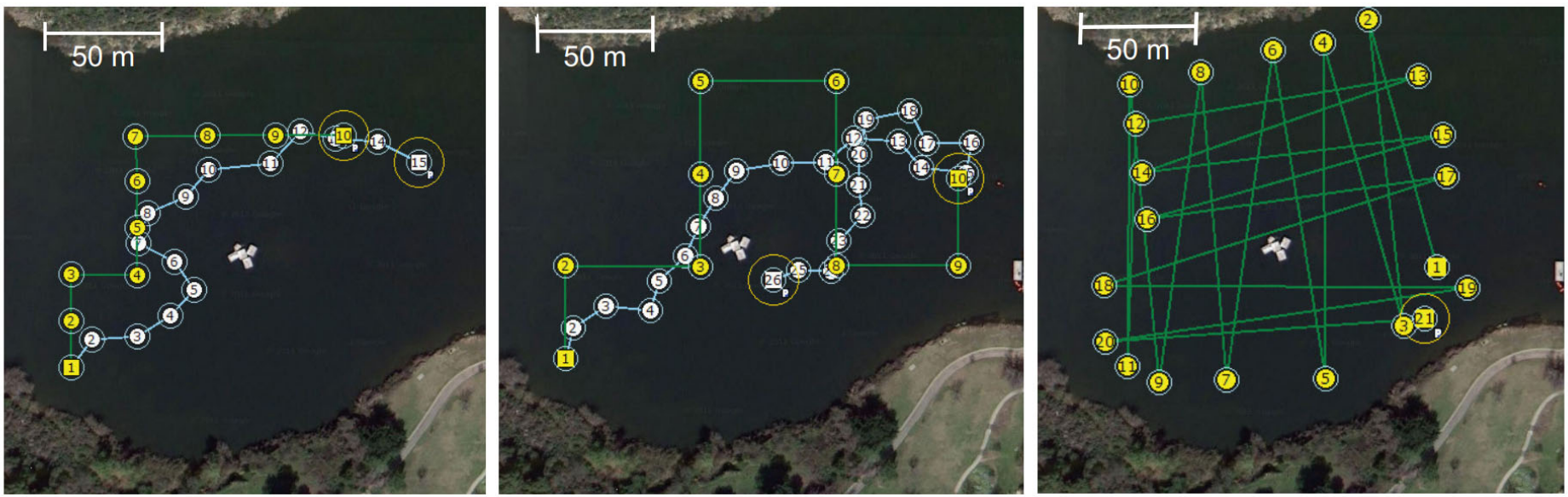
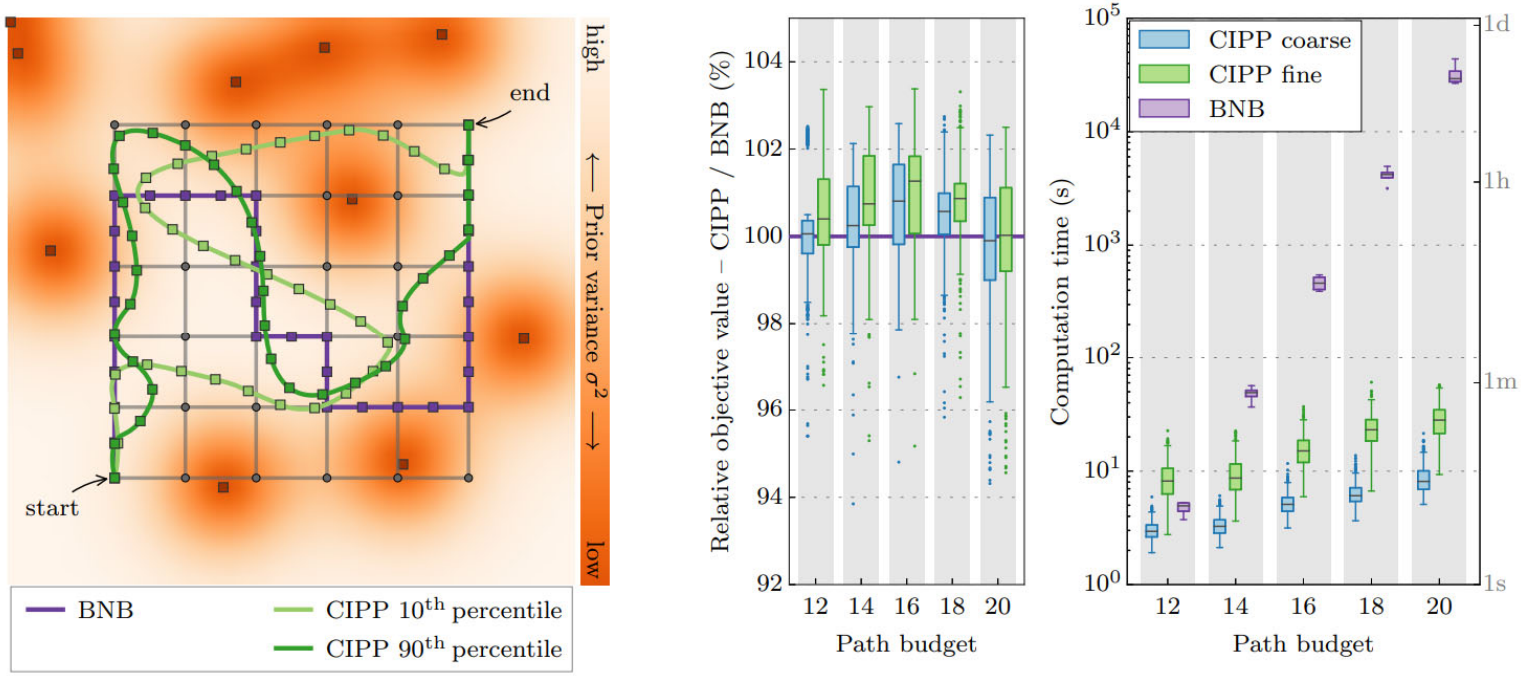


(a) Budget = 200 m (b) Budget = 400 m (c) Full survey = 2500 m

Hollinger and Sukhatme (2014), "Sampling-based robotic information gathering algorithms," in: The IJRR. 33(9): 1271-1287.

Planning Strategies

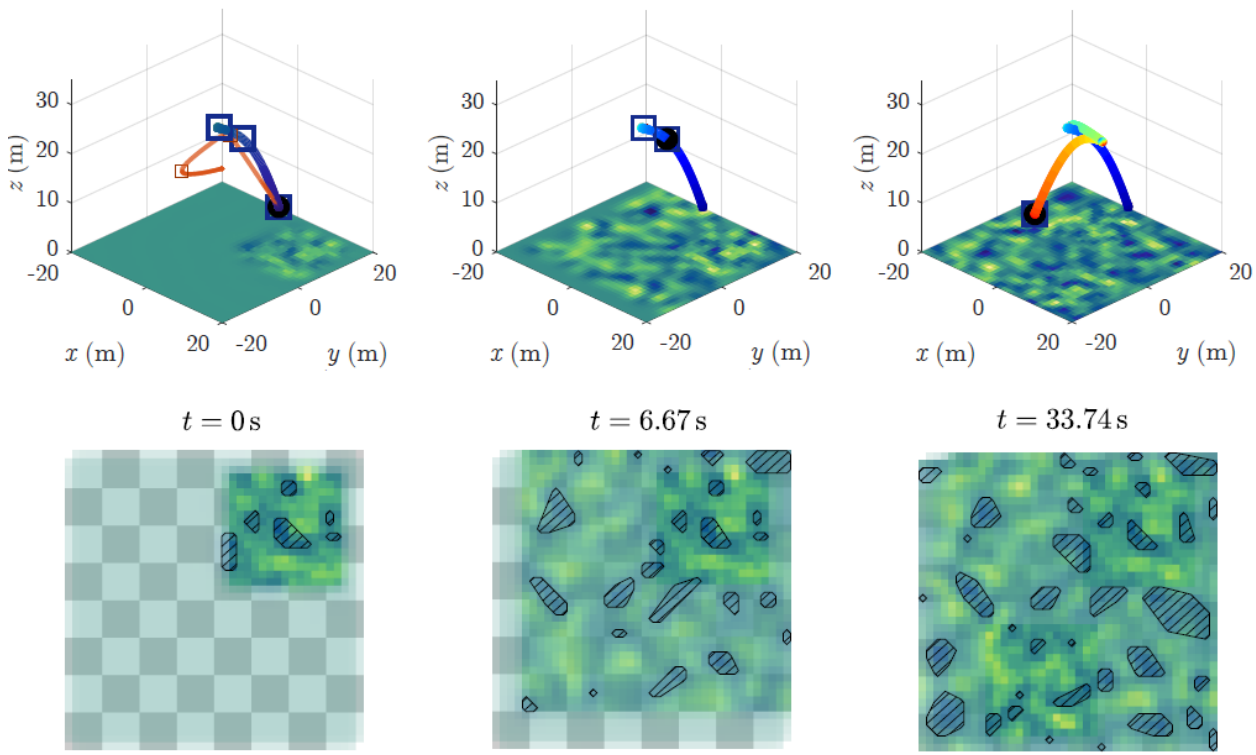
- Continuous-space methods
  - Trajectory optimisation: continuous informative path planning (CIPP) for lake monitoring
  - Evolutionary optimisation technique (CMA-ES)



Hitz et al. (2017), "Adaptive continuous-space informative path planning for online environmental monitoring," in: Journal of Field Robotics. 34(8): 1427-1449.

Planning Strategies

- Continuous-space methods
  - Trajectory optimisation for agricultural monitoring
  - Evolutionary optimisation technique



Popović et al. (2020), “An informative path planning framework for UAV-based terrain monitoring,“ in: Autonomous Robots. 44: 889-911..

Planning Strategies

- Continuous-space methods
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Popović et al. (2020), “An informative path planning framework for UAV-based terrain monitoring,“ in: Autonomous Robots. 44: 889-911..

Planning Strategies

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  - Trajectory optimisation for agricultural monitoring
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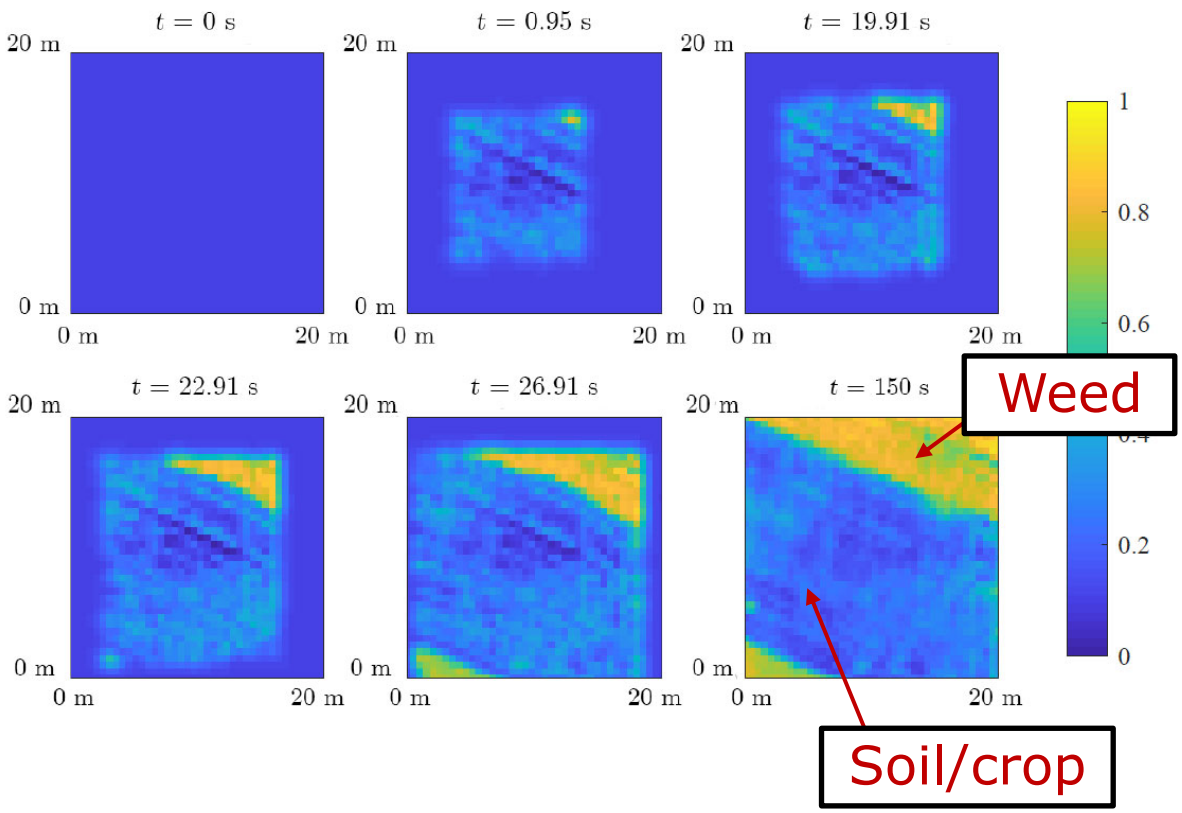

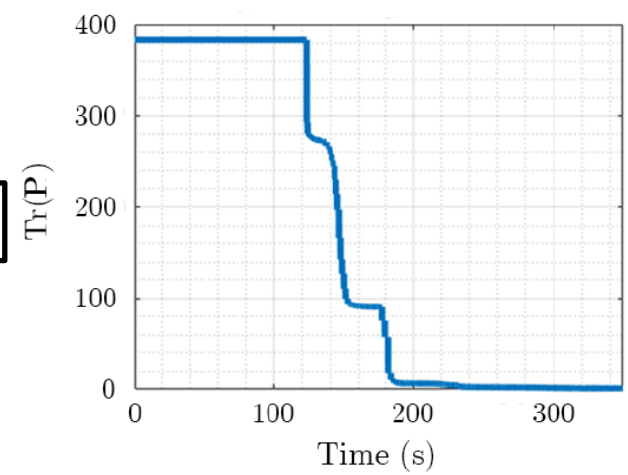

Popović et al. (2020), "An informative path planning framework for UAV-based terrain monitoring," in: Autonomous Robots. 44: 889-911..

Summary

- Robotic information gathering
  - Motivation
  - Environmental mapping, utility calculation, planning algorithm
- Robotic exploration
  - Frontier-based exploration and improvements
- Robotic data collection
  - Informative planning in Gaussian Processes

Further Reading

- Bai et al. (2021), “Information-Driven Path Planning,“ in: Current Robotics Reports. 2: 177-188.
- Lluvia et al. (2021). “Active Mapping and Robot Exploration: A Survey,” in: Sensors. 21(7): 2445.
- Krause et al. (2008), “Near-Optimal Sensor Placements in Gaussian Processes,“ in: Jour. of Machine Learning Research. 9: 235-284.
- Kernel Cookbook (toronto.edu)
- Microsoft PowerPoint - submodularity-slides.ppt (ethz.ch)

Thank you for your attention